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Multidimensional wavelet neural networks based on polynomial powers of sigmoid: a framework to image verification

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Abstract Wavelet functions have been used as the activation function in feed forward neural networks. An abundance of R&D has been produced on wavelet neural network area. Some successful algorithms and applications in wavelet neural network have been developed and reported in the literature. However, most of the aforementioned reports impose many restrictions in the classical back propagation algorithm, such as low dimensionality, tensor product of wavelets, parameters initialization, and, in general, the output is one dimensional, etc. In order to remove some of these restrictions, a family of polynomial wavelets generated from powers of sigmoid functions is presented. We described how a multidimensional wavelet neural networks based on these functions can be constructed, trained and applied in pattern recognition tasks. As examples of applications for the method proposed a framework for face verification is presented.

Keywords Artificial neural network, Human face verification, Image processing, Pattern recognition, Polynomial powers of Sigmoid (PPS), Wavelets.

Introduction

Wavelet functions have been successfully used in many problems as the activation function of feed forward neural networks. There are claims that many biological fundamental properties can emerge from wavelet transformation in Marar (1997). An abundance of R&D has been produced on wavelet neural network area. Some successful algorithms and applications in wavelet neural network have been developed and reported in the literature Zhang and Benveniste (1992); Marar (1997); Oussar and Dreyfus (2000); Chen and Hewit (2000); Zhang and San (2004); Fan and Wang (2005); Zhang and Pu (2006); Chen et al. (2006); Avci (2007); Jiang et al. (2007); Misra et al. (2007).

However, most of the aforementioned reports impose many restrictions in the classical back propagation algorithm, such as low dimensionality, tensor product of wavelets, parameters initialization, and, in general, the output is one dimensional, etc.

In order to remove some of these restrictions, we develop a robust Three Layer PPS-Wavelet multi-dimensional strongly similar to classical Multilayer Perceptron. The great advantage of this new approach is that PPS-Wavelets offers the possibility choice of the function that will be used in the hidden layer, without need to develop a new learning algorithm. This is a very interesting property for the design of new wavelet neural networks architectures. This paper is organized as follows. Section "Function approximation" covers basic theoretical aspects in function approximation. Section "Wavelet functions" introduces the wavelet sigmoidal function. Section "Polynomial powers of Sigmoid" presents the framework used in this research. Section "Human face verification" deals with application of face verification problem.

Function approximation

Multilayer perceptron networks (MLP) have been intensely studied as efficient tools for arbitrary function approximation. Amongst the developments achieved in the theory of function approximation using MLP, the work carried out by Hecht-Nielsen resulted in an improved version for the superposition theorem defined by Sprecher in Hecht-Nielsen (1987). Galant and White in 1988 showed that a feed forward network with one hidden layer of processing units that use flat cosines as the activation function correspond to a special case of Fourier networks that can approximate a Fourier series for a given function. Cybenko developed a rigorous demonstration that MLPs with only one hidden layer of processing elements is sufficient to approximate any continuous function with support in a hypercube by Cybenko (1989).

The theorem is directly applied to MLP. The sigmoid, radial basis and wavelets functions are a common choice for the network construction since it satisfies the conditions imposed in the theorem. The theorem of function approximation provides a mathematical basis that gives support

to the approximation of any continuous arbitrary function. Furthermore, it defines for the case of MLP that a network composed of only one hidden layer neurons is sufficient to compute, in a given problem, a mapping from the input space to the output space, based on a set of training examples. However, with respect to training speed and ease of implementation, the theorem does not provide any insight about the solutions developed. The choice of activation functions and the learning algorithm defines which particular network is used. In any situation, the neurons operate as a set of functions that generate an arbitrary basis for function approximation which is defined based on the information extracted from the input-output pairs. For training a feed forward network, the back propagation algorithm is one of the most frequently employed in practical applications and can be seen as an optimization.

Wavelet functions

Two categories of wavelet functions, namely, orthogonal wavelets and wavelet frames (or non-orthogonal), were developed separately by different interests. An orthogonal basis is a family of wavelets that are linearly independent and mutually orthogonal, this eliminates the redundancy in the representation. However, orthogonal wavelets bases are difficult to construct because the wavelet family must satisfy stringent criteria in Daubechies (1992); Chui (1992). This way, for these difficulties, orthogonal wavelets is a serious draw-back for their application to function approximation and process modeling in Oussar and Dreyfus (2000). Conversely, wavelet frames are constructed by simple operations of translation and dilation of a single fixed function called the mother wavelet, which must satisfy conditions that are less stringent than orthogonality conditions.

Let φ_j a wavelet, the relation:

$$\varphi_j(\mathbf{x}) = \varphi(d_j \cdot (\mathbf{x} - t_j))$$

Where t_j are the translations factors and d_j is the dilation factors $\in \mathbb{R}$. The family of functions generated by \mathcal{U} can be defined as:

$$\mathcal{U} = \{\varphi(d_j \cdot (x - t_j)), t_j \text{ and } d_j \in \mathbb{R}\}$$

A family \mathcal{U} is said to be a frame of $L^2(\mathbb{R})$ if there exist two constants $c > 0$ and $C < \infty$ such that for any square integrable function f the following inequalities hold:

$$c \|f\|^2 \leq \sum_j |\langle \varphi_j, f \rangle|^2 \leq C \|f\|^2$$

Where $\varphi_j \in \mathcal{U}$, $\|f\|$ denotes the norm of function f and $\langle \varphi_j, f \rangle$ the inner product of functions. Families of wavelet frames of $L^2(\mathbb{R})$ are univer-

sal approximators in Zhang and Benveniste (1992); Pati and Krishnaprasad (1993). In this work, we will show that wavelet frames allow practical implementation of multidimensional wavelets. This is important when considering problems of large input and output dimension. For the modeling of multi-variable processes, such as, the artificial neural networks biologically plausible, multidimensional wavelets must be defined. In the present work, we use multidimensional wavelets constructed as linear combination of sigmoid, denominated Polynomial Powers of Sigmoid Wavelet (PPS-wavelet).

Sigmoidal wavelet functions

In Funahashi (1989) is showed that:

Let $s(x)$ a function diferent of the constant function, limited and monotonically increase. For any $0 < \alpha < \infty$ the function created by the combination of sigmoid is described in Equation 1:

Equation 1

$$g(x) = s(x + \alpha) - s(x - \alpha)$$

where $g(x) \in L^1(\mathbb{R})$, i.e,

$$\int_{-\infty}^{\infty} g(x) < \infty$$

in particular, the sigmoid function satisfies this property.

Using the property came from the Equation 1, in Pati and Krishnaprasad (1993) boundary suggests the construction of wavelets based on addition and subtraction of translated sigmoidal, which denominates wavelets of sigmoid. In the same article show a process of construction of sigmoid wavelet by the substitution of the function $s(x)$ by $Y(qx)$ in the Equation 1. So, the Equation 2 is the wavelet function created in Pati and Krishnaprasad (1993).

Equation 2

$$\psi(x) = g(x + r) - g(x - r)$$

where $r > 0$. By terms of sigmoid function, the Equation 2, $\psi(x)$ is given by:

Equation 3

$$\psi(x) = Y(qx + a + r) - Y(qx - a + r) - Y(qx + a - r) + Y(qx - a - r)$$

where $q > 0$ is a constant that control the curve of the sigmoid function and α and $r \in \mathbb{R} > 0$.

Pati and Krishnaprasad demonstrated that the function $\psi(x)$ satisfies the admissibility condition for wavelets by Daubechies (1992); Chui (1992). The Fourier Transform of the function $\psi(x)$ is given by the Equation 4:

Equation 4
$$\int_{-\infty}^{\infty} \psi(x)e^{-iwx} dx = -i \frac{4\pi \sin(x\alpha)\sin(wr)}{q \sinh\left(\frac{\pi w}{q}\right)}$$

In particular, we accepted for analysis and practical applications the family of sigmoid wavelet generated by the parameters $q = 2$ and $\alpha = r$,

Equation 5
$$\psi(x) = Y(2x + m) - 2Y(2x) - Y(2x - m)$$

as example. So, the Equation 3 can be rewritten the following form:

where $m = \alpha + r$.

Following, partially, this research line, we present in the next section a technique for construction of wavelets based on linear combination of sigmoid powers.

Polynomial powers of Sigmoid

The Polynomial Powers of Sigmoid (PPS) is a class of functions that have been used in recent years to solve a wide range of problems related to image and signal processing in Marar (1997). Let $Y: \mathbb{R} \rightarrow [0,1]$ be a sigmoid function defined by $Y(x) = 1/(1+e^{-x})$. The n^{th} -power of the sigmoid function is a function

$$Y^n : \mathbb{R} \rightarrow [0,1] \text{ defined by } Y^n(x) = \left(\frac{1}{1+e^{-x}}\right)^n$$

Let Θ be a set of all power of functions defined by (6):

Equation 6
$$\Theta = \{Y^0(x), Y^1(x), Y^2(x), \dots, Y^n(x), \dots\}$$

An important aspect is that the power these functions, still keeps the form of the letter S. Looking the form created by the power functions of sigmoid, suppose that the n^{th} power of the sigmoid function to be represented by the following form:

Equation 7
$$Y^n(x) = \frac{1}{a_0 + a_1 e^{-x} + a_2 e^{-2x} + \dots + a_n e^{-nx}}$$

where $a_n, a_1, a_2, \dots, a_n$ are some integer values. The extension of the sigmoid power can be viewed like lines of a Pascal's triangle. The set of function written by linear combination of polynomial powers of sigmoid is defined as PPS function. The degree of the PPS is given by the biggest power of the sigmoid terms.

Polynomial wavelet family on PPS

The derivative of a function $f(x)$ on $x = x_0$ is defined by:

$$f'(x_0) = \lim_{\Delta x \rightarrow 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$$

since the limits there is. So, if we do the computation of the Equation 8:

Equation 8
$$\frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$$

for a small value of Δx , showed have a good approximation for $f'(x_0)$. Naturally, Δx can be positive or negative. So, if is we use negative value for Δx , the expression will be:

Equation 9
$$\frac{f(x_0 - \Delta x) - f(x_0)}{-\Delta x}$$

This way, we can say that the arithmetic measure of the Equations 8 and 9 will be a good approximation for $f'(x_0)$ too. Then, we can write the following Equation 10:

Equation 10
$$f'(x_0) \simeq \frac{f(x_0 + \Delta x) - f(x_0 - \Delta x)}{2\Delta x}$$

By convenience, we consider $p = 2\Delta x$ and its substitution in the Equation 10. So, we have the Equation 11:

Equation 11
$$f'(x_0) \simeq \frac{f\left(x_0 + \frac{p}{2}\right) - f\left(x_0 - \frac{p}{2}\right)}{p}$$

this point we computed an approximated value for the second derivative of $f(x)$ in $x = x_0$. From the Equation 11, changing $f(x)$ by $f'(x)$, we obtain the Equation 12:

Equation 12
$$f''(x_0) \simeq \frac{f'\left(x_0 + \frac{p}{2}\right) - f'\left(x_0 - \frac{p}{2}\right)}{p}$$

reusing the Equation 11, we can write:

$$f'\left(x_0 + \frac{p}{2}\right) \simeq \frac{f(x_0 + p) - f(x_0)}{p}$$

and

$$f' \left(x_0 - \frac{p}{2} \right) \simeq \frac{f(x_0) - f(x_0 - p)}{p}$$

using these results in the Equation 12, we have an approximation of the second derivative of $f(x)$ in $x = x_0$ that is given by:

Equation 13

$$f''(x_0) \simeq \frac{f(x_0 + p) - 2f(x_0) + f(x_0 - p)}{p^2}$$

The approximation given by the Equation 13 is extremely adequate for the $f(x)$ is a sigmoid function. Suppose that $f(x)$ is a sigmoid, for example, $Y(x)$. So, the second derivative of $Y(x)$ is approximated by the Equation 14:

Equation 14

$$Y''(x_0) \simeq \frac{Y(x_0 + p) - 2Y(x_0) + Y(x_0 - p)}{p^2}$$

Due the fact of the sigmoid function to be continuous and differentiable for any $x \in \mathbb{R}$, we can say that the Equation 14 is true for any x_0 , then we can write the Equation 15, defined for all $x \in \mathbb{R}$.

Equation 15

$$Y''(x) \simeq \frac{Y(x_0 + p) - 2Y(x_0) + Y(x_0 - p)}{p^2}$$

Comparison the Equations 15 and 5, we do there analysis for the approximation of the second derivative of sigmoid function. The first for values of $p \geq 1$ and the second for values of $p < 1$.

Case $p \geq 1$:

It is clear that the function given by the sigmoid second derivative approximation, Equation 15, also will have the same form of the Pati and Krishnaprasad functions, except of a p^2 constant that divides their amplitude. So, the following result is true: when $p > 1$ always there is a sigmoid wavelet which integral of the admissibility condition by Daubechies (1992); Chui (1992) limited the same integral of the Equation 15. Therefore, the approximation of the second derivative of the sigmoid function is a wavelet too.

Case $p < 1$:

In this case, we will analyze when p is going to zero, i.e.,

Equation 16

$$\lim_{p \rightarrow 0} \frac{Y(x_0 + p) - 2Y(x_0) + Y(x_0 - p)}{p^2}$$

this limit tends to the second derivative of the function is given on PPS terms by:

Equation 17
$$\varphi_2 = 2Y^3(x) - 3Y^2(x) + Y(x)$$

where we denominated $\varphi_2(x)$ the first wavelet the sigmoid function. The others derivatives, begin on the second, we considered true by derivative property by Fourier Transform in Marar (1997). The successive derivation process of sigmoid functions, allowed joining a family of wavelets polynomial functions. Among many applications for this family of PPS-wavelets, special one is that those functions can be used like activation functions in artificial neurons. The following results correspond to the the analytical functions for the elements $\varphi_3(x)$ and $\varphi_4(x)$ that are represented by:

$$\begin{aligned} \varphi_3 &= -6Y^4(x) + 12Y^3(x) - 7Y^2(x) + Y(x) \\ \varphi_4 &= 24Y^5(x) - 60Y^4(x) + 50Y^3(x) - 15Y^2(x) + Y(x) \end{aligned}$$

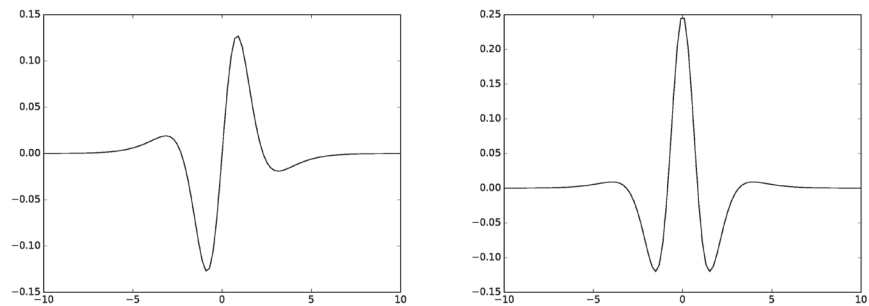


Figure 1 PPS-wavelets examples, φ_4 and φ_5

Estimating the coefficients of PPS-wavelets

Considering j the number of wavelets that are to be defined, the algorithm below calculates a matrix of integer values that estimates the coefficients of the PPS-wavelets.

Step 1: Initialization

$$\begin{aligned} C_{1,1} &\leftarrow 1; \\ C_{1,2} &\leftarrow 1; \end{aligned}$$

The initial values are considered only auxiliary variables. The matrix of value associated with the process of wavelet construction is obtained from the second row.

Step 2: Calculate the coefficient of the PPS of the highest degree

$$\begin{aligned} n &\leftarrow 3; \\ n &\leftarrow n + 1; (n \leq j) \\ C_{n-1,n} &\leftarrow C_{n-2,n-1} * (n - 1) * (-1)^{n+1}; \end{aligned}$$

Step 3: Calculate the coefficients of the remaining terms of the polynomial

$$\begin{aligned} k &\leftarrow n; \\ k &\leftarrow k - 1; (k > 2) \\ C_{n-1,k-1} &\leftarrow C_{n-2,k-1} * (k - 1) + C_{n-2,k-2} * (k - 2) * (-1)^k; \end{aligned}$$

Step 4: Calculate the coefficients of the first power variable

$$C_{n-1,1} \leftarrow 1$$

It is important to notice that steps 2 and 3 are cascaded by an inherent dependence on variable n . By proceeding in above way, a family of polynomial wavelets are generated.

PPS Wavelet neural network

Let us consider the canonical structure of the multidimensional PPS-wavelet neural network (PPS-WNN), as shown in Figure 2.

For the PPS-WNN in Figure 2, when a input pattern $X=(x_1,x_2,\dots,x_m)^T$ is applied at the input of the network, the output of the i^{th} neuron of output layer is represented as a function approximation problem, i.e., $f: R^m \rightarrow [0,1]^n$, given by:

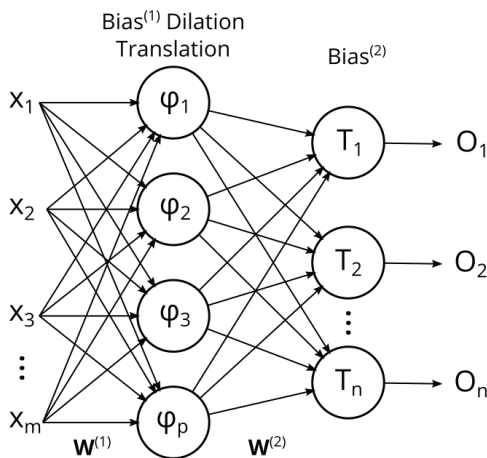


Figure 2 PPS-wavelet neural network Architectures

$$O_i(x) \approx Y_i \left(\sum_{j=1}^p w_{ij}^{(2)} \varphi_j \left(d_j \cdot \left(\sum_{k=1}^m w_{jk}^{(1)} x_k - b_j^{(1)} \right) - t_j \right) - b_i^{(2)} \right)$$

where p is number of hidden neurons, $Y(\cdot)$ is sigmoid function, $\varphi(\cdot)$ is the PPS-wavelet, $w^{(2)}$ are weight between the hidden layer to the output layer, $w^{(1)}$ are weights between the input to the hidden layer, d are dilation factors and t are translation factors of the PPS-wavelet, $b^{(1)}$ and $b^{(2)}$ are bias factors of the hidden layer and output layer, respectively.

The PPS-WNN contains PPS-wavelets as the activation function in the hidden layer (Figure 3) and sigmoid function as the activation function in the output layer (Figure 4).

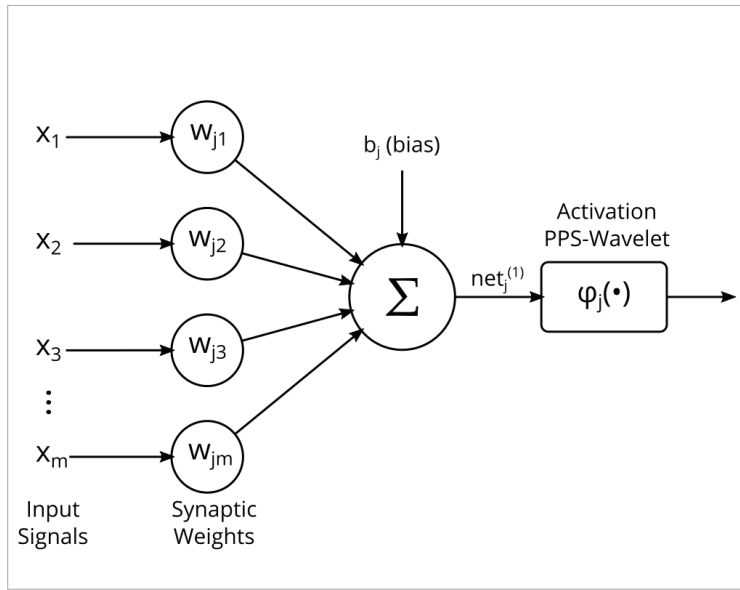


Figure 3 The Hidden Neuron of PPS-Wavelet Neural Network

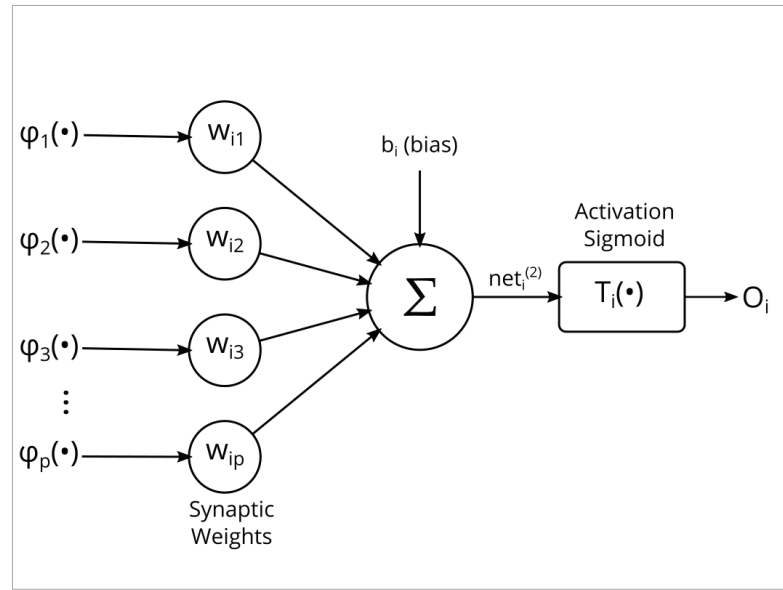


Figure 4 The Output Neuron of PPS-Wavelet Neural Network

The output of the j^{th} PPS-wavelet hidden neuron (Figure 3) is given by:

$$\odot_j = \varphi_j(d_j \cdot (net_j^{(1)} - t_j))$$

where

$$net_j^{(1)} = \sum_{k=1}^m w_{jk}^{(1)} x_k - b_j^{(1)}$$

The output of the i^{th} output layer neuron (Figure 4) is given by:

$$\odot_i = \frac{1}{1 + \exp(-net_i^{(2)})}$$

where

$$net_i^{(2)} = \sum_{j=1}^p w_{ij}^{(2)} \varphi_j(d_j \cdot (net_j^{(1)} - t_j)) - b_i^{(2)}$$

The adaptive parameters of the PPS-WNN consist of all weights, bias, translations and dilation terms. The sole purpose of the training phase is to determine the “optimum” setting of the weights, bias, translations and dilation terms so as to minimize the difference between the network output and the target output. This difference is referred to as training error

of the network. In the conventional back propagation algorithm, the error function is defined as:

Equation 19

$$E = \frac{1}{2} \sum_{q=1}^s \sum_{i=1}^n (y_{qi} - o_{qi})^2$$

where the n is the dimension of output space, s is the number of training input patterns.

The most popular and successful learning method for training the multilayer perceptrons is the back propagation algorithm. The algorithm employs an iterative gradient descent method of minimization which minimizes the mean squared error (L^2 norm) between the desired output (y_i) and network output (o_i). From Equations (18) and (19), we could deduce the partial derivatives of the error to each PPS-wavelet neural network parameter's, which is given by:

Partial equations of the output layer:

Equation 20

$$\frac{\partial E}{\partial w_{ij}^{(2)}} = - \sum_{q=1}^s (y_{qi} - o_{qi}) \cdot o_{qi} \cdot (1 - o_{qi}) \cdot \varphi_j(d_j \cdot (net_{qj}^{(1)} - t_j))$$

Equation 21

$$\frac{\partial E}{\partial b_i^{(2)}} = - \sum_{q=1}^s (y_{qi} - o_{qi}) \cdot o_{qi} \cdot (1 - o_{qi})$$

Partial equations of the hidden layer:

Equation 22

$$\frac{\partial E}{\partial w_{jk}^{(1)}} = -d_j \sum_{q=1}^s \left[(\varphi'_j(d_j \cdot (net_{qj}^{(1)} - t_j))) \cdot x_{qk} \cdot \sum_{i=1}^n (y_{qi} - o_{qi}) \cdot o_{qi} \cdot (1 - o_{qi}) \cdot w_{ij}^{(2)} \right]$$

Equation 23

$$\frac{\partial E}{\partial b_j^{(1)}} = - \sum_{q=1}^s \left[(\varphi'_j(d_j \cdot (net_{qj}^{(1)} - t_j))) \cdot d_j \cdot \sum_{i=1}^n (y_{qi} - o_{qi}) \cdot o_{qi} \cdot (1 - o_{qi}) \cdot w_{ij}^{(2)} \right]$$

Partial equations of the pps-wavelet parameters:

Equation 24

$$\frac{\partial E}{\partial d_j} = \sum_{q=1}^s \left\{ \left[(\varphi'_j(d_j \cdot (net_{qj}^{(1)} - t_j))) \cdot (net_{qj}^{(1)} - t_j) \right] \cdot \sum_{i=1}^n (y_{qi} - o_{qi}) \cdot o_{qi} \cdot (1 - o_{qi}) \cdot w_{ij}^{(2)} \right\}$$

Equation 25

$$\frac{\partial E}{\partial t_j} = - \sum_{q=1}^s \left[(\varphi'_j(d_j \cdot (net_{qj}^{(1)} - t_j))) \cdot d_j \cdot \sum_{i=1}^n (y_{qi} - o_{qi}) \cdot o_{qi} \cdot (1 - o_{qi}) \cdot w_{ij}^{(2)} \right]$$

After computing all partial derivatives the network parameters are updated in the negative gradient direction. A learning constant γ defines the step length of the correction, r is the iteration and momentum factor is β . The corrections are given by:

$$w_{ij}^{(2)}(r+1) = w_{ij}^{(2)}(r) - \gamma \cdot \frac{\partial E}{\partial w_{ij}^{(2)}} + \beta \cdot (w_{ij}^{(2)}(r) - w_{ij}^{(2)}(r-1))$$

$$b_i^{(2)}(r+1) = b_i^{(2)}(r) - \gamma \cdot \frac{\partial E}{\partial b_i^{(2)}} + \beta \cdot (b_i^{(2)}(r) - b_i^{(2)}(r-1))$$

$$w_{jk}^{(1)}(r+1) = w_{jk}^{(1)}(r) - \gamma \cdot \frac{\partial E}{\partial w_{jk}^{(1)}} + \beta \cdot (w_{jk}^{(1)}(r) - w_{jk}^{(1)}(r-1))$$

$$b_j^{(1)}(r+1) = b_j^{(1)}(r) - \gamma \cdot \frac{\partial E}{\partial b_j^{(1)}} + \beta \cdot (b_j^{(1)}(r) - b_j^{(1)}(r-1))$$

$$d_j(r+1) = d_j(r) - \gamma \cdot \frac{\partial E}{\partial d_j} + \beta \cdot (d_j(r) - d_j(r-1))$$

$$t_j(r+1) = t_j(r) - \gamma \cdot \frac{\partial E}{\partial t_j} + \beta \cdot (t_j(r) - t_j(r-1))$$

Algorithm to PPS wavelet neural network

In this section, the learning algorithm to the PPS-wavelet neural network is proposed by using the back propagation method. Where the initialization procedures, attribute random values on [0,1] to the parameters. However, improvements in the initialization process have been proposed by the selection of basic functions PPS-wavelet in de Queiroz and Marar (2007).

```

Begin
initialize-choice-PPS-function();
initialize-architecture();
initialize-weights();
initialize-PPSwavelet-neurons-dilatations();
initialize-PPSwavelet-neurons-translations();
initialize-neurons-bias();

Do-While ( $epoch \leq epoch_{max}$ ) or ( $\frac{1}{2}total_{error} > acceptable_{error}$ )
BeginDo-While
totalerror ← 0;
randomize-input-patter-order();

For pattern counter  $q = 1..$ 
Beginfor
read input pattern  $x_{(q,j)}: j = 1..m$ 
read input target vector  $y_{q,i}: i = 1..n$ 
acc-param-h-layer(); by Eqs. (22) – (25)
compute  $O_{(q,i)}$  by Eq. (18)
acc-param-o-layer(); by Eqs. (20) – (21)
totalerror ← totalerror +  $(y_{(p,k)} - O_{(p,k)})^2$ 
Endfor

IF ( $total_{error} > acceptable_{error}$ ) Then
Beginthen
update-param-o-layer();
update-param-h-layer()
Endthen

epoch ← epoch + 1
EndDo-While
End

```

Human face verification

Systems based on biometric characteristics, such as face, fingerprints, geometry of the hands, iris pattern and others have been studied with attention. Face verification is a very important of these techniques because through it nonintrusive systems can be created, which means that people can be computationally identified without their knowledge. This way, computers can be an effective tool to search for missing children, suspects or people wanted by the law. Mathematically speaking, human face verification problem can be formulated as function approximation problems and from

the viewpoint of artificial neural networks these can be seen as the problem of searching for a mapping that establishes a relationship from an input to an output space through a process of network learning.

This study presents a system for detection and extraction of faces based on the approach presented in Lin and Fan (2001), which consists of finding isosceles triangles in an image, as the mouth and eyes form that geometric figure when linked by lines. In order for these regions to be determined, the images must be converted into binary images, thus the vertices of the triangles must be found and a rectangle must be cut out around them so that their size can be brought to normal and the area can be fed into a second part of the system that will analyze whether or not it is a real face. Three different approaches are tested here: A weighing mask is used to score the region, proposed by Lin and Fan (2001), a classical MLP back propagation (MLP-BP) and PPS-wavelet neural network, for the analysis to be performed.

Image treatment

First the image was read with the purpose of allocating a matrix in which each cell indicates the level of brightness of the correspondent pixel; then, it is converted into a binary matrix by means of a Threshold parameter T , because the objects of interest in our case are darker than the background. This stage changes to 1 (white) a brightness level greater than T and to 0 (black). In most of the cases, due to noise and distortion in the input image, the result of the binary transformation can bring a partition image and isolated pixels. Morphologic operations - opening followed by closing - are applied with the purpose of solving or minimizing this problem by Gonzalez and Woods (2002). The Figure 8 shows the result of these operations.

Segmentation of potential face regions

After binarization the task is finding the center of three 4-connected components that meet the following characteristics:

- ¶ vertex of an isosceles triangle by Lin and Fan (2001);
- ¶ the Euclidean distance between the eyes must be 90-100 % the distance between the mouth and the central point between the eyes by Lin and Fan (2001);
- ¶ the triangle base is at the top of the image.

The last restriction does not allow finding upside down faces, but it significantly reduces the number of triangles in each image, thus reducing the processing time to the following stages. For example, the numbers of triangles found in Figure 5 (D), with this restriction 399 and without restriction 769.

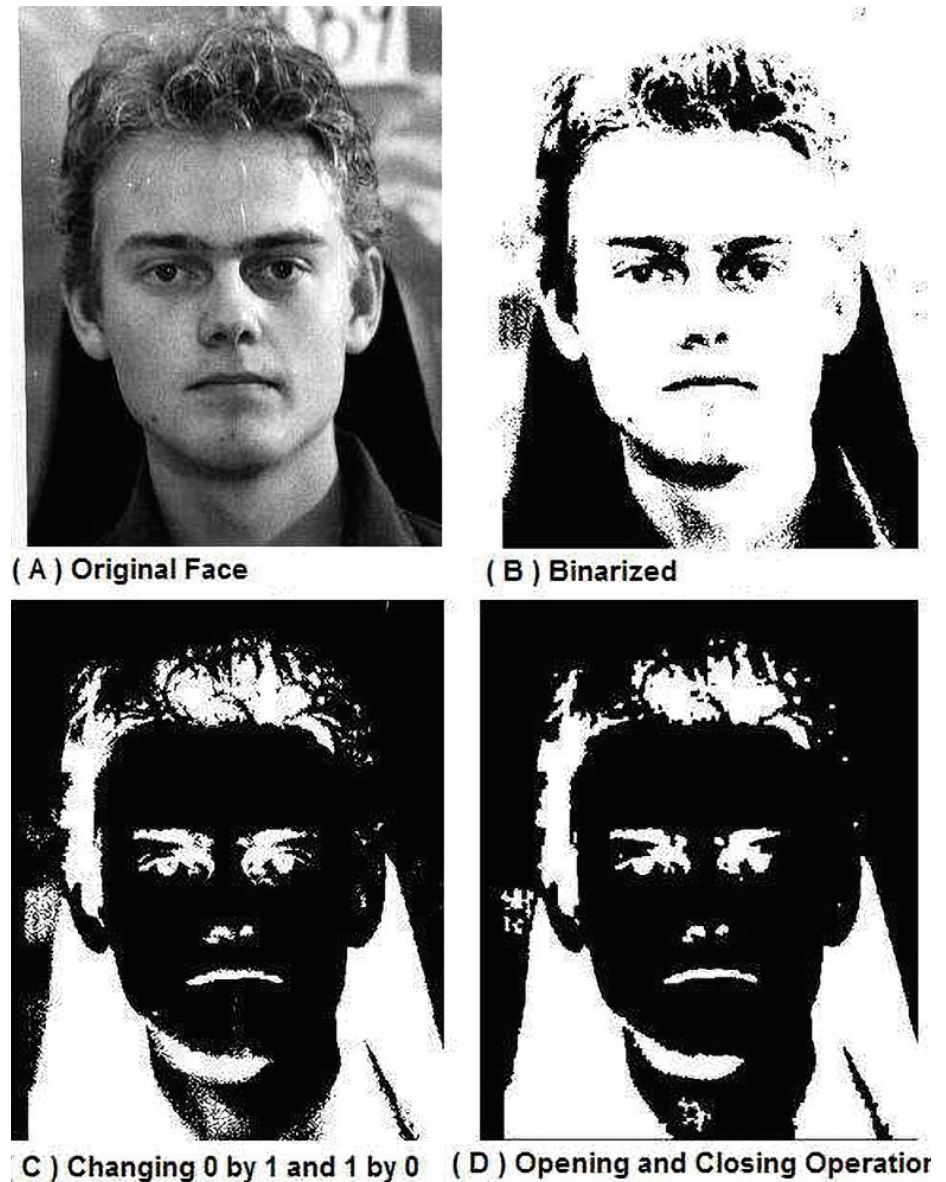


Figure 5 Image treatment after morphologic operations

The opening and closing operations are vital, since it is impossible to determine the triangles without this image treatment. The processing mean time to find the results presented was 4 seconds; on the other hand, 8 hours were insufficient in an attempt at finding the same results using a Pentium 4 with 2.4 Ghz processor in Figure 5 (C).

Normalization of potential facial regions

Once the potential face regions that we have selected in the previous section are allowed to have different sizes. All regions had to be normalized to the (60x60) pixels size by bi-cubic interpolation technique, because every potential regions needs to present the same amount of infor-

mation for comparison. So, normalization of a potential region can reduce the effects of variation in distance and location.

Face's pattern recognition

The purpose of this stage is to decide whether a potential face region in an image (the region extracted in the first part of the process) actually contains a face. To perform this verification, two methods were applied: The weighting mask function, described by Lin and Fan (2001) and PPS-wavelet neural network.

The weighting mask function

The function Weighting Mask, according to the author, it is based on the following idea: If the normalized potential region is really contains a face, it should have high similarity to the mask that is formed by 10 binary training faces (Mask Generation). Every normalized potential facial region is applied into the weighting mask function that is used to compute the similarity between the normalized potential facial region and the mask. The computed value can be used in deciding whether a potential region contains a face or not.

Mask generation

The mask was created using 10 images. The first five are pictures of females and the others are pictures of males. All of them were manually segmented, binarized, normalized, morphologically treated (opening and closing) and then the sum of the correspondent cell of each image was stored in the 11th matrix. Finally, that matrix was binarized with another Threshold T, for which values lower than or equal to T were replaced by 0, and the others by 1. The result was improved with T=4. Whereas at lower values the areas of the eyes and mouth become too big, at higher values these areas almost disappear. In both cases, determining the triangles is considerably difficult.

Weighting mask algorithm

```
Begin  
Input the region R and mask M; p=0;  
For all pixels of R and M  
  IF the pixel from R and M is white  
    Then p = p+6;  
  IF the pixel from R and M is black  
    Then p = p+2;  
  IF the pixel from R is white and that from M is black  
    Then p = p - 4;  
  IF the pixel from R is black and that from M is white  
    Then p = p - 2;  
End
```


The algorithm used to decide whether a potential face (R) contains a real face is based on the idea that the binary image of a face is highly similar to that of the mask.

A set experimental results demonstrates that the threshold values should be a set between face $3400 \leq p \leq 6800$ by Marar et al. (2004).

PPS-wavelet neural network

In order to demonstrate the efficiency of the proposed model. Two PPS-WNNs, one with the activation function $\varphi_2(\cdot)$ and the other with $\varphi_5(\cdot)$ in the hidden layer, were implemented to analyze when a potential face region really contains a face. However, the raw data face, (60 x 60) pixels, cannot be used directly for the training the networks because the features are deep hidden. Therefore, we used the Principal Components Analysis (PCA) method to create a face space that represents all the faces using a small set of components Marar (1997). For this purpose we consider the first 15 components as the extracted features or face space. In that case study, 100 manually segmented faces (50 women and 50 men) and more 40 non-face random images were used to network training.

Therefore, the PPS-WNNs and classical MLP-BP architectures with 15 units in the input layer, with 16 PPS-wavelet neurons in the hidden layer and with 2 neurons in the output layer were designed and trained. Here, in the output layer, we represented face by the vector (1, 0) and non-face by the vector (0, 1). We used, as test, the same regions (R) applied to the previous method.

Face verifications results

Several tests were performed to determine an ideal threshold value for the conversion of the images into binary figures. In a scale from 0 (black) to 1 (white), 0.38 was empirically determined as a good value to most of the images, but to darker images 0.22 was a better value. The test was done through the use of 100 images (50 male and 50 female) with two different threshold values from Department (2003). The results are shown in Table 1.

	Threshold	0.22	0.38
Weighting Mask	Correct Detection	81 %	48 %
	False Detection	25 %	21 %
Classical MLP-BP	Correct Detection	83 %	35 %
	False Detection	28 %	17 %
PPS-WNN $\varphi_2(\cdot)$	Correct Detection	85 %	63 %
	False Detection	15 %	23 %
PPS-WNN $\varphi_5(\cdot)$	Correct Detection	92 %	51 %
	False Detection	5 %	11 %

Table 1 Face verification results with 2 threshold values

Conclusion

Neural networks and wavelet transform have been recently seen as attractive tools for developing efficient solutions for many real world problems in function approximation. The combination of neural networks and wavelet transform gives rise to an interesting and powerful technique for function approximation referred to as wavenets. Function approximation is a very important task in environments where computation has to be based on extracting information from data samples in the real world processes. So, mathematical model is a very important tool to guarantee the development of the neural network area.

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